Discrete shedding modes of the cylinder wake in a jet with a homogeneous core

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The von Kármán vortex street behind a circular cylinder in a laminar homogeneous jet core is experimentally investigated. The Strouhal–Reynolds number relationships are measured for various shear-layer thicknesses and aspect ratios. The experimental Strouhal number values are found to collapse with the discrete vortex shedding modes, which were observed for boundary-layer end conditions. The results indicate that the shedding modes are independent from the end conditions, but are an intrinsic feature of the shedding process of an infinitely long cylinder. The experimentally assumed shedding mode, however, is strongly dependent of the geometric parameters, like the shear-layer thickness and the aspect ratio. The observed tendencies can be made physically plausible.

I. INTRODUCTION

The wake of a circular cylinder can be characterized by a staggered array of vortices, the so-called *von Kármán vortex street.* In this paper, the vortex street of a cylinder with shear layers as end conditions is investigated. In particular, the validity of the discrete shedding modes given for boundary-layer end conditions by König, Eisenlohr, and Eckelmann;[1] König, Noack, and Eckelmann;[2] and König[3] is examined.

The characteristic property of the cylinder wake is given by the dimensionless velocity, the Reynolds number $\mathrm{Re}=ud/\nu$, u being the mean velocity, d is the diameter of the cylinder, and ν is the kinematic viscosity of the fluid. The temporal periodicity of the vortex street is described by the dimensionless frequency, the Strouhal number $\mathrm{St}=fd/u$, where f represents the shedding frequency.

The vortex street is formed by vortices that separate alternately from both sides of the cylinder. For Reynolds numbers smaller than 170, oblique shedding of vortex tubes with a shedding angle θ to the cylinder axis is observed. For Reynolds numbers larger than 170, a complex vortex pattern with a dominant temporal periodicity is formed. The laminar array of vortex tubes is observed in a Reynolds number range of $40<\mathrm{Re}<170$; this range is called *regular.* For $170<\mathrm{Re}<300$, the vortex pattern is three dimensional. This *transitional range* is characterized by an intermittent change of laminar and small-scaled three-dimensional shedding. For $300<\mathrm{Re}$, the vortex street is spatially and temporally irregular.

Detailed measurements of the shedding frequency of the vortices were carried out by Roshko.[4] He found two different Strouhal–Reynolds number relationships for the regular Reynolds number range and for $300<\mathrm{Re}$. In contrast to the continuous Roshko law for the laminar vortex street, Tritton[5] observed a discontinuity in the Strouhal–Reynolds number relationship. Above $\mathrm{Re}=90$, the Strouhal number was reported to be significantly lower than the corresponding value from Roshko's law. Tritton attributed this discontinuity to two different two-dimensional shedding processes. The discovery of this discontinuity, the so-called *Tritton jump,* induced many investigations of the discontinuous behavior of the laminar vortex street.

Gaster[6,7] investigated sections of a cone in homogeneous flow and a circular cylinder in inhomogeneous flow. He concluded that the Tritton discontinuities for the laminar vortex street of a cylinder could result from three-dimensional effects, even for small variations of the local Reynolds number. Berger and Wille,[8] however, confirmed Tritton's results, particularly the existence of two different shedding processes, called *high-* and *low-speed mode.*

Gerich and Eckelmann[9] found a spanwise variation of the Strouhal number behind a cylinder in homogeneous flow. The Strouhal number was 10%–15% lower in the end regions of the cylinder if endplates were mounted. This region was called *end cell.* König, Eisenlohr, and Eckelmann[1] observed that the end conditions of the cylinder also have a strong influence on the shedding frequency in the central region. The Tritton jump was conclusively identified as an end effect. The boundary layer at the endplate causes an oblique shedding of the vortex tubes, not only at the end regions of the cylinder, but also in the central region. The oblique shedding was observed to reduce the shedding frequency, as compared to parallel shedding. Williamson[10] found experimentally that the Strouhal number depends only on the shedding angle θ, according to: $\mathrm{St}_\theta=\mathrm{St}_0\cos(\theta)$, where St_θ and St_0 represent the Strouhal numbers of oblique and parallel shedding. By manipulating the end conditions also, a parallel shedding in the regular Reynolds number range can be enforced (Williamson;[11] Eisenlohr and Eckelmann;[12] and Hammache and Gharib[13]).

The shedding angle and the Strouhal number were expected to vary continuously with changing the end conditions. For the laminar vortex street, König *et al.*[1] observed

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that a variation of the boundary-layer thickness at the ends of the cylinder causes different Strouhal numbers in the midspan region for a given Reynolds number. Varying the end conditions, they found only a few Strouhal–Reynolds number relations. König, Noack, and Eckelmann[2] could show for a large variety of boundary-layer thicknesses and aspect ratios that the Strouhal number does not depend continuously on the end conditions. The Strouhal number can only assume discrete values given by a scheme of Strouhal–Reynolds number relations, called *modes*:

$$\mathrm{St}_n(\mathrm{Re})=\mathrm{St}_0(\mathrm{Re})-n[\mathrm{St}_0(\mathrm{Re})-\mathrm{St}_1(\mathrm{Re})],$$

where the integral non-negative index n denotes the mode number. Here, $n=0$ corresponds to parallel shedding and $n>0$ for oblique shedding. The index n increases with the shedding angle.

This mode scheme proposed by König *et al.*[2] was verified for boundary layers as end conditions of the cylinder. In the present paper we will investigate the validity of the Strouhal-number modes for shear layers as end conditions and the variation of the mode number with the shear-layer thickness, the Reynolds number, and the aspect ratio of the cylinder.

In Sec. II we describe the experimental setup used for the investigation. In Sec. III, the experimental results are presented, and in Sec. IV the physical phenomena are summarized and discussed.

II. EXPERIMENTAL SETUP

In order to realize shear-layer end conditions, the cylinder is placed in a jet of an open return wind tunnel, i.e., the tunnel ends in a circular nozzle with an outlet diameter of $D=180$ mm. The nozzle was specially designed to achieve laminar homogeneous flow in the core of the jet for low free-stream velocities, particularly less than 3 m/s (Brede, Ohle, and Eckelmann[14]).

The cylinders are made from stainless steel and have a polished surface. Their diameter d ranges from 0.5 to 2 mm. They are mounted perpendicular to the flow with the supporting points outside of the jet, and are mechanically decoupled from the wind tunnel. The distance x of the cylinder to the nozzle is varied from $0.25D$ to $1.06D$. All cylinders are under mechanical tension to prevent vibrations. In addition, the thinnest cylinder of a diameter of 0.5 mm is damped against transversal vibrations by pressing rubber pistons at the cylinder near the mounting points. Under these conditions, no vibrations are present.

The velocity is varied from 0.3 to 9 m/s depending on the cylinder diameter, yielding Reynolds numbers from 40 to 300. The spanwise variation of the velocity within the jet core is lower than 0.5%. As an example, the velocity profile in the shear-layer region for $x=0.53D$ is shown in Fig. 1 for a centerline velocity $u_{\mathrm{cl}}=1$ m/s. In this figure, the local velocity u is normalized by u_{cl} and the spanwise coordinate z by D. The shear-layer thickness s is characterized by the tangent at the inflection point of the velocity profile and the intercepts with the lines $u/u_{\mathrm{cl}}=0$, and $u/u_{\mathrm{cl}}=1$. Here s is normalized with D. Table I compiles the shear-layer parameters for $u_{\mathrm{cl}}=1$ m/s. The shear-layer rolls up before the cylinder position, thus giving rise to fluctuations with a characteristic frequency that is roughly two orders of magnitudes smaller than the typical shedding frequencies, i.e., no resonance phenomena can be expected. The corresponding velocity fluctuations are located in the shear layer, while the laminar core is steady within the experimental resolution. Hence, the von Kármán vortex street and the shear layer are dynamically decoupled.

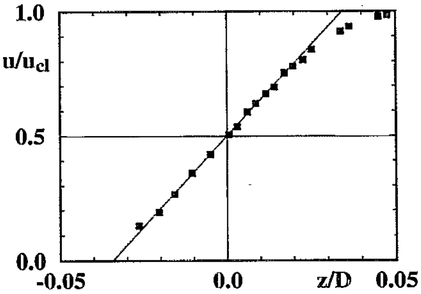

FIG. 1. Velocity profile of the shear layer for $s/D=0.077$. The corresponding distance from the nozzle exit is $x/D=0.53$.

A hot-wire probe is placed $6d$ downstream of the cylinder and $2d$ off the center of the wake to measure the characteristic frequency. The measurements are made in the midspan plane. A digital FFT analyzer is used to obtain the shedding frequency automatically from the spectrum of the velocity fluctuations. The flow velocity is monitored by a Prandtl tube, connected to an MKS baratron pressure transducer.

Further details about the experimental setup can be inferred from Brede.[15]

III. EXPERIMENTAL RESULTS

The experimentally realized Strouhal numbers depend on three characteristic numbers, the Reynolds number and two geometric numbers for the end conditions, the nondimensionalized shear-layer thickness s/d, and the aspect ratio l/d, where l represents the section of the cylinder within the laminar core of the jet. For all considered sets of parameters, our experimental Strouhal number values lie always very close to one of the discrete modes of König *et al.*,[2] which were found for a cylinder with boundary layers as end conditions. Hence, the experimental value of the Strouhal number can more conveniently be described by the mode number n of the corresponding mode.

Figure 2 shows the Strouhal number in terms of the Reynolds number for various aspect ratios and constant s/d. The

TABLE I. Characteristic parameters of the shear layer for a flow speed of 1 m/s. Here x represents the distance from the nozzle exit, z is the spanwise coordinate, s is the shear-layer thickness, u is the local velocity, and D is the diameter of the nozzle.

x/D	s (mm)	s/D	$(du/dz)_{\max}$ (s^{-1})
0.25	7.7	0.043	129.9
0.53	13.6	0.077	73.5
1.06	34.9	0.190	28.7

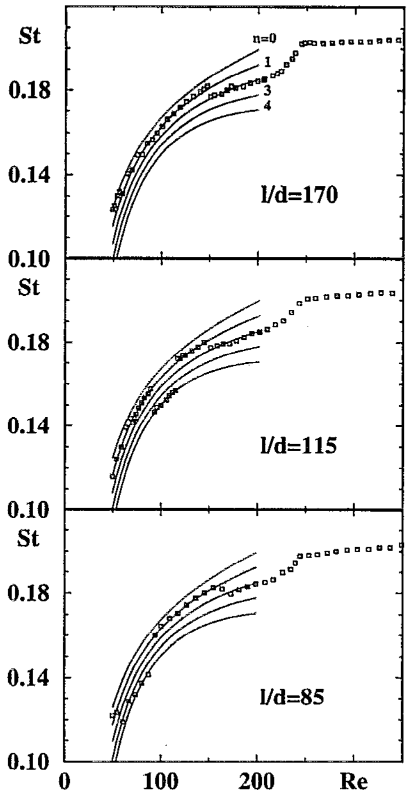

FIG. 2. Strouhal–Reynolds-number relationships for various aspect ratios l/d (decreasing from top to bottom) and a shear-layer thickness of approximately $s/d=7$, which slightly varies with l/d.

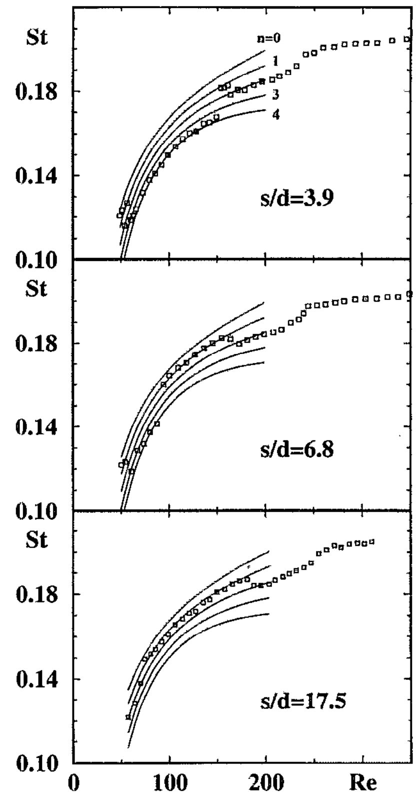

FIG. 3. Strouhal–Reynolds-number relationships for various shear-layer thicknesses s/d (increasing from top to bottom) and an aspect ratio of approximately 80, which slightly varies with s/d.

Strouhal number is found to be independent of the aspect ratio for Re>170. This result agrees well with prior observations of König[3] and König and Eckelmann[16] for boundary-layer end conditions. The Strouhal number depends, however, on the aspect ratio for Re<170. In general, the mode number corresponding to the observed Strouhal number values increases with decreasing aspect ratio. For large aspect ratios, the preferred mode numbers are $n=0$ and $n=1$, whereas for smaller aspect ratios $n=4$ is found. The largest mode number is generally observed for Reynolds numbers around 100. For an aspect ratio of 115, mode $n=4$ is embedded in an interval of mode $n=1$. For an aspect ratio of 85, mode $n=4$ is found at and somewhat below a Reynolds number of 100.

In Fig. 3, the Strouhal–Reynolds number dependency is shown for various shear-layer thicknesses and constant aspect ratio. The bottom part of Fig. 2 is duplicated in the second subfigure of Fig. 3 for reasons of comparison. The Strouhal number is independent of the shear-layer thickness for Re>170. For Re<170, the mode number tends to decrease with increasing s/d. Here, mode $n=4$ dominates for small s/d and mode $n=1$ for large s/d. The Reynolds number interval corresponding to mode $n=1$ extends from Re $=160$ toward lower Reynolds numbers.

In each of the graphs of Figs. 2 and 3, the two geometric parameters are fixed. To summarize the results of all parameter variations, mode charts with only one fixed parameter are created. These charts are obtained from interpolated three-dimensional grid values for $n(\mathrm{Re}, l/d, s/d)$. The values for n of intermediate points in the grid are linearly interpolated from the experimental results for n and rounded to the nearest integer.

Figure 4 displays three mode charts for three constant shear-layer thicknesses. From each chart the mode number can be inferred as functions of the Reynolds number and the aspect ratio. The grey tones illustrate the mode number in the (Re, l/d, s/d) space. The small-scaled patterns in the mode chart are artifacts from the interpolation process. For Re >170, the mode number n varies insignificantly with both geometric parameters, s/d and l/d. In the regular range Re <170, however, two general tendencies are observed. First, n increases with decreasing aspect ratio. Second, n decreases with increasing shear-layer thickness. In particular, an interval containing mode $n=4$ is observed at Re$\approx$100 for low aspect ratios and high shear-layer thicknesses. In this Reynolds number interval, the mode number is typically lower than the one in the adjacent regions. This holds also for aspect ratios and shear-layer thicknesses, which are larger than the parameter ranges described above. These general tendencies result in a convergence of the mode number to medium values with increasing shear-layer thickness. Instead of the mode numbers $n=0$ and $n=4$, the intermediate mode numbers $n=1$ and $n=2$ are observed; i.e., thin shear layers with large velocity gradients induce the largest and the lowest observed modes $n=0$ and $n=4$.

The four mode charts shown in Fig. 5 illustrate the value

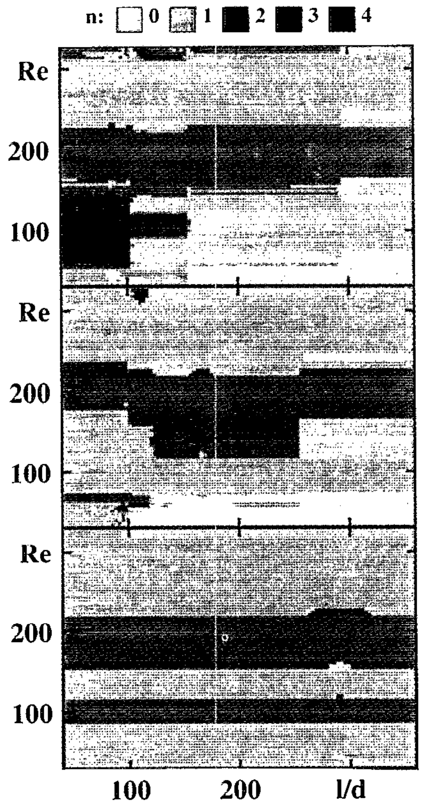

FIG. 4. Mode number n in terms of the aspect ratio l/d and the Reynolds number Re for $s/d=4$, 14, and 70 (top to bottom).

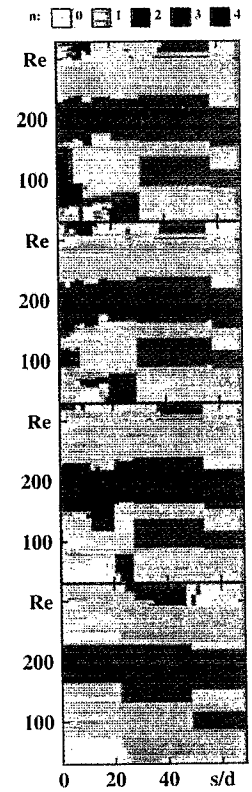

FIG. 5. Mode number n in terms of the shear-layer thickness s/d and the Reynolds number Re for aspect ratios l/d of 85, 115, 170, and 345 (top to bottom).

of $n(\mathrm{Re}, l/d, s/d)$ for different aspect ratios l/d. From these charts, the mode number is displayed as a function of shear-layer thickness and Reynolds number. This illustration confirms the general tendencies of Fig. 4. The main variation of the mode number with the shear-layer thickness, and the aspect ratio occurs only in the regular Reynolds number range. Particularly for low shear-layer thicknesses, the mode number decreases with increasing aspect ratio. An increase of the shear-layer thickness results in intermediate mode numbers 1 or 2. Mode numbers 0 and 4 are only observed for a thin shear layer.

IV. CONCLUSIONS

For the first time, a systematic investigation of the von Kármán vortex street has been carried out with shear layers as end conditions. The Strouhal number as a function of the Reynolds number, the aspect ratio, and the shear-layer thickness is determined. For all considered sets of parameters, the experimental Strouhal number values collapse with the shedding modes of König *et al.*,[2] which were found for boundary-layers end conditions. Our results are further evidence that the discreteness of the shedding modes is independent of the end conditions of the cylinder, but is an intrinsic feature of the vortex shedding behind an infinitely long cylinder.

The experimentally observed discretization is unexpected because generally the flow properties depend continuously on the stationary boundary conditions. The solutions of the initial-boundary-value problem of the Navier–Stokes equation is typically implicitly assumed to be well posed. This implies that the solutions vary continuously with the boundary conditions. Our experiments, however, show discrete jumps of the Strouhal number values in terms of the geometric parameters. This indicates that there must exist some, so far unexplored, physical discretization mechanism that is inherently different from resonance phenomena.

The experimentally assumed mode number has been investigated for various Reynolds numbers, aspect ratios, and shear-layer thicknesses. For the mode number, no simple dependency can be evidenced. Nevertheless, there are few general tendencies that can be made physically plausible. The lowest and so far highest observed mode numbers $n=0$ and $n=4$, respectively, occur for thin shear layers. These modes are a consequence of the large velocity gradient du/dz within the shear layer of the jet. For high aspect ratios, the large gradient seems to decouple the end region from the central part of the wake, and thus has a similar effect as slanted end plates (Williamson[10]), end cylinders (Eisenlohr and Eckelmann[12]), or front cylinders (Hammache and Gharib[13]). In the central region, this yields low mode numbers, even down to $n=0$, i.e., parallel shedding. For low aspect ratios, large spanwise velocity gradients induce a strong deformation of the von Kármán vortices, which results in an oblique shedding with an extreme angle (mode numbers up to $n=4$).

The largest mode numbers for fixed geometric parameters have been found in a Reynolds number interval around 100, whereas the mode number was observed to be smaller in the neighboring intervals. For low Reynolds numbers, viscosity effects in spanwise direction have more time to enforce parallel shedding. Hence, the mode number is small for low Reynolds numbers. At higher Reynolds numbers, more angular momentum oriented in spanwise direction is produced in the boundary layer of the cylinder. This induces stronger von Kármán vortices that try to remain parallel to the cylinder for reasons of momentum preservation. This might explain the low mode numbers observed at Reynolds numbers somewhat below 160. For the intermediate Reynolds-number range, the effects of viscosity and momentum preservation are impaired, and the end effects seem to govern the separation angle of the vortices. This process results in large mode numbers up to 4.

Although we can make the general tendencies plausible, the following unresolved question remains. Does there exist a physical principle for the quantization of vortex shedding just like there exists principles for the quantization of the discrete energy eigenvalues of the Schrödinger equation for the hydrogen atom?

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