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S. Narayanan
United Technologies Research Center
East Hartford, CT, U.S.A.

B.R. Noack
Hermann-Föttinger-Institute of Fluid Mechanics,
Technische Universität Berlin, Berlin, GERMANY

E. Meiburg
University of California at Santa Barbara,
Santa Barbara, CA, U.S.A.

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Satish Narayanan*
United Technologies Research Center, E. Hartford, CT 06108, U.S.A.

Bernd R. Noack
Hermann-Föttinger-Institute of Fluid Mechanics, Technische Universität Berlin, Berlin, GERMANY

Eckart H. Meiburg†
*Department of Mechanical and Environmental Engineering, University of California at Santa Barbara,
Santa Barbara, CA 93106, U.S.A.*

ABSTRACT

The flow and sound associated with the near field of a three-dimensional, unsteady jet flow are modeled using a moderate-dimensional vortex model, namely using inviscid, incompressible vortex filament simulations. The model captures the dynamics and sound generation from organized motion in the turbulent jet flow field, neglecting the fine-scale structure, viscous and compressibility effects. The jet flow model simulations are used to reproduce the mean and unsteady characteristics of the jet flow measured experimentally in a turbulent, M_j=0.6, cold, single stream jet. A low Mach number acoustic analogy is used to compute the noise source distribution and radiation associated with the jet. Sound generation associated with the organized flow structures and their interactions in the jet is investigated. Good comparisons of the spatial distribution of jet noise sources with experimental measurements are obtained, suggesting promise for the use of such dynamical models in the analysis and control of jet noise.

NOMENCLATURE

c	Speed of sound
D	Diameter of nozzle
f	Frequency
G	Power spectral density
M	Mach number
Re	Reynolds number
St	Strouhal number, f D /U
u’	Turbulence intensity
U	Longitudinal velocity
p	Far-field acoustic pressure
ω	vorticity
ϕ	velocity potential

Subscripts

j	Jet exit condition
uu	Longitudinal velocity component fluctuation
ref	reference condition

INTRODUCTION

The noise generation from high subsonic jet exhausts is of interest to commercial air transport, particularly in light of increasingly stringent community noise regulations. Methods to achieve jet noise reduction rely on extensive and expensive experimental testing. Clearly a better understanding of the noise generation mechanisms and development of better models would facilitate the design of successful noise reduction concepts. Significant research has been undertaken in understanding and controlling noise generation, but a model-based prediction approach has remained elusive (e.g. see Tam[1]). In particular, models for noise generation associated with the (controllable) dynamics of large-scale, coherent, unsteady flow structures are absent. Experiments are plagued by limitations in spatiotemporal resolution with which turbulence statistics can be measured (e.g. see Bridges & Podboy[2]). On the other hand, “high fidelity” numerical simulations such as direct numerical simulations (DNS) and large eddy simulations (LES) are restricted to low Reynolds number jet flows and simple geometries[3,4]. Semi-empirical models of jet noise generation, using steady Reynolds Averaged Navier Stokes simulations (RANS), have been somewhat capable of predicting the far field acoustic signature of the jet[5,6]. However, tuning of the turbulence models is required and the applicability may be restricted to noise generation from disorganized, fine-scale turbulence. The noise predictions are typically limited to sideward angles, and the computation of noise radiation at aft angles (where jet noise is dominant) is poor. In particular, these models do not provide a systematic approach for developing noise reduction concepts, since fine-scale turbulence features are not typically controllable using large scale (passive/active) devices

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*Research Engineer, Member AIAA
†Professor

(which are better suited for controlling large-scale organized structures). While organized flow structures, found to be prevalent in high Reynolds number turbulent flows, are believed to also play a significant role in noise generation[7], models describing or capable of predicting the associated sound are yet unavailable. The use of methods such the proper orthogonal decomposition provide a posteriori low-dimensional description of the jet flow field and are in their formative stages, but reduced-order models have not emerged[8,9].

A clear need is therefore emerging for dynamical models which can capture the noise generation from coherent flow motion, and which can be used to propose and evaluate noise reduction concepts. The present study uses a three-dimensional vortex model to simulate the noise generation arising from organized/coherent structures in jet flows. This “low fidelity” modeling approach attempts to capture the essential organized flow features and their noise generation physics in a simple analytical/computational framework with which control studies can be performed. The models therefore do not provide quantitative predictions. Such “non-traditional” modeling is in contrast to recent computational methods for jet noise, where the focus has been on developing accurate noise prediction methodology. As a first step, we have initiated the vortex-based modeling of the jet for the initial development region, prior to the end of the jet potential core. Note that although the peak noise generation occurs farther downstream near the end of the jet potential core, the EPNL-sensitive high frequencies (relevant to noise regulations) occur within the first few jet diameters[10], hence our interest in modeling the unsteady flow physics and noise generation in this region. Following detailed analysis and validation for these flow models, it is expected that modeling of larger flow domains will be feasible.

This study aims to provide a reduced order modeling approach for a three-dimensional jet flow and its noise generation. In the following we: (i) develop a reduced order model for simulating the organized, unsteady motion underlying a turbulent jet; (ii) validate the flow model with experimental data; (iii) analyze the sound generation from the flow model, using a simplified analytical approach; and (iv) validate the acoustics analysis and provide some insights into the noise generation process.

MODELING AND NUMERICAL METHOD: VORTEX FILAMENT SIMULATIONS

In recent years, three-dimensional vortex dynamics simulations have become a powerful tool for simulating the evolution of the dominant, coherent structures that are typically observed in high Reynolds number turbulent shear flows. Vortex methods are well suited to explore flow regimes that are not accessible via other numerical approaches, which suffer from Reynolds number limitations (e.g. DNS), very high computational costs (e.g. LES), or do not allow the analysis of time dependent effects (e.g. RANS). Consequently, the use of vortex dynamics techniques has resulted in significant contributions towards an improved understanding of the dynamics of temporally and spatially evolving mixing layers, wakes, and jets, with extensions to many other applications.

The main attraction of vortex methods lies in their Lagrangian nature, which in inviscid flows preserves the identity of rotational fluid elements. This allows the tracking of individual vortex lines or tubes, making vortex dynamics methods highly suitable for exploring control strategies, since the consequences of time-dependent local actuation can be tracked throughout the flow field. Furthermore, the Lagrangian nature of these methods minimize the effects of numerical dissipation, since no spatial derivatives are evaluated. This allows the simulation of high Reynolds number flows in which the dominant structures evolve in a nearly inviscid fashion. Great computational savings can be realized when employing vortex dynamics methods, since only the rotational (vortical) part of the flow field has to be discretized. In free shear flows, this is typically a small section of the overall flow domain. As a result, even fairly complex simulations typically consume on the order of a day of CPU time on a reasonably powerful workstation. In grid-based, Eulerian methods, the entire flow domain needs to be discretized.

Extensive efforts have been undertaken over the years to validate vortex dynamics techniques. Ashurst[11] simulated a spatially growing, two-dimensional mixing layer via a vortex method, obtaining good agreement with experimental measurements for the streamwise velocity profiles, the rms-velocity fluctuations and Reynolds stresses. The comparisons are described in reviews of three-dimensional vortex dynamics techniques by Leonard[12] and Meiburg[13].

Inviscid vortex dynamics simulations, like inviscid grid-based calculations, cannot properly represent viscosity-driven processes such as dissipation, the reconnection of vortex tubes, or the generation of vorticity at no-slip boundaries. In recent years, extensions have been developed to accurately incorporate the effects of viscosity into vortex dynamics methods, so that now DNS via vortex methods is feasible. However, these viscous vortex dynamics variants suffer from similar disadvantages as grid-based Eulerian DNS techniques, in that they cannot reach the high Reynolds number regime due to prohibitive resolution requirements.

Two-dimensional and axisymmetric vortex models have been applied to the noise generation from free shear flows[14]. These studies have demonstrated that vortex models can capture organized aspects of the unsteady jet flow fields, and have been coupled with an acoustic analogy to compute the noise. Qualitative features of the far field acoustics have been somewhat successfully computed, but the noise sources and their distribution have not been investigated. Several advances have been made in the use of inviscid, incompressible vortex methods, making the computations more robust, accurate and realistic[15]. We build on recent advances in three-dimensional vortex methods, combined with a previously developed low Mach number acoustic analogy, for developing and validating a new vortex filaments-based approach for modeling sound generation from a three-dimensional jet flow. The experimental flow field and acoustics results used for comparisons here have been reported in detail elsewhere[10,16].

For the jet simulations, we have extended an existing code for the temporally evolving problem (of an incompressible, axisymmetric mixing layer) to that for a spatially evolving flow. The spatially evolving simulation requires that we extend the domain over several streamwise wavelengths. In the streamwise direction, the flow is typically discretized into O(100) filaments. Each filament initially contains on the order O(20-30) segments in the circumferential direction. These parameters were determined by refining the discretization until a further increase in resolution resulted in very small changes in the mean and fluctuating characteristics of the jet flow (discussed further later). The inviscid vorticity transport equation is solved and the velocity field is computed using the Biot-Savart integration (see eq. 1), which is carried out with second order accuracy both in space and in time by employing the predictor-corrector time-stepping scheme, with the trapezoidal rule for the spatial integration.

$$\mathbf{u}(\mathbf{x}) = \frac{1}{4\pi}\iiint d\mathbf{x}' \frac{\omega(\mathbf{x}') \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} + \nabla\phi \qquad (1)$$

As the flow structure evolves, the vortex filaments undergo considerable stretching. To maintain an adequate resolution, the cubic spline representation of the filaments can be used to introduce additional nodes, based on a criterion involving distance and curvature[15]. Furthermore, the time-step is repeatedly reduced as local acceleration effects increase. In a laminar or transitional flow, the filament core radius decreases as its arc length increases to conserve its volume. For the turbulent jet, however, the unresolved turbulent entrainment and dissipation effects are modeled by a core-expansion model. The vortex-filament volume increases linearly in streamwise direction at a rate approximately half the natural shear layer growth. Moreover, since our interest is in the computation of the "large" scales, fine-scale features farther downstream can be spatially averaged. The flow and acoustic computations were done on an UltraSPARC60 workstation and typically had run times on the order of 12-24 hours, depending on the number of nodes used to discretize the filaments.

Good predictions of the growth rates for the axial Kelvin-Helmholtz instability of the jet were used to demonstrate the accuracy and convergence of the vortex filament simulations[17]. In order to render the problem dimensionless, we take the streamwise velocity difference between the centerline and infinity as the characteristic velocity. The thickness of the axisymmetric shear layer serves as the characteristic length scale.

FLOW FIELD MODEL SIMULATIONS

The vortex filament model focuses on the initial flow region within the first four jet diameters, with an additional exit domain length of one jet diameter in which the "smooth" departure of large-scale vortical structures is ensured by accelerating the combination of filaments. Furthermore, fixed filaments for an additional jet diameter upstream of the nozzle exit plane were used to ensure a top hat profile jet flow. **Figure 1** displays a snapshot of the Lagrangian vortex filament field, showing naturally developing three dimensionality on the vortex filaments confined to the jet shear layer, with axisymmetric filaments being released at the location designated as the nozzle exit. No "noise" was added at the inlet boundary, and the disturbances for the jet evolution are from small numerical discretization errors.

Sensitivity studies of model parameters. The key parameters associated with the vortex filament computation are the core size of filament nodes to eliminate singularities, and the rate of core expansion to mimic diffusion. Mean velocity and turbulence intensity profiles and shear layer velocity spectra measured in the near field of the $M_j \cong 0.6$ jet[16] were used to validate the model parameters. The initial core size of the filament nodes were set in line with the measured vorticity thickness in the boundary layer at the nozzle exit of the turbulent, M_j=0.6 jet. The filament release frequency is derived from the core size of the filament nodes and an average convection speed of half the centerline jet velocity. This frequency is verified to be at least a couple of orders of magnitude higher than that associated with the flow structures downstream, thus minimizing the potential contamination of the large-scale vortex dynamics by numerical errors.

The filament release frequency, the filament initial core size and its streamwise expansion rate were varied to determine the sensitivity of the flow and noise source computations to the model parameters. Although the filament node core size to jet diameter ratio used here is nearly three times more than that measured experimentally, the simulated flow evolution and that measured experimentally were found to be similar, thus obviating the need for further refinement of the core size (requiring higher dimensional simulations). The exit boundary layer azimuthal vorticity thickness was estimated from the experimentally measured mean velocity profile. The filament node core size and its release frequency govern the spatial discretization of the flow field. The overlap of filament nodes in the streamwise and azimuthal directions allowed for these two parameters to be controlled independently. Detailed studies were conducted to ensure that further model refinement (e.g., halving of the filament node core size or doubling of the filament release frequency) did not noticeably affect the qualitative features of the flow and the more sensitive noise source distributions. Another critical parameter is the filament node core size expansion (noted earlier), which affects the streamwise growth of mean and fluctuating characteristics of the jet. Very aggressive expansion of the filament cores causes drastic dampening of unsteadiness (even jet re-laminarization) with lower turbulence intensities compared to experimental data and faster decaying velocity spectra (similar to RANS-type turbulence models). Very low levels of expansion on the other hand produce a highly unsteady jet flow with much larger turbulence levels compared to experimental data, presumably because of the lack of the damping action of turbulent eddy viscosity in realistic turbulent jets. Based on these sensitivity studies an acceptable set of model parameters were chosen, while maintaining a moderate dimensional vortex filament model (with O(100) filaments for streamwise discretization and O(30) nodes for azimuthal discretization).

Model-based flow simulations. The longitudinal components of the mean velocity and turbulence intensity profiles recorded at x/D_j=1 match well with measurements for a M_j=0.6 jet[16], as shown in **Fig. 2(a,b)**, with similar distribution and levels. The power spectral density associated with the fluctuating component of the longitudinal velocity is shown in **Fig. 3(a)**, revealing broadband spectral peaks associated with the passage of coherent vortical structures in the jet shear layer. The velocity time traces were recorded radially along the inner edge of the jet shear layer at two streamwise locations, x/D_j=1,2. **Figure 3(b)** displays the velocity spectra from experimental measurements at similar radial and streamwise locations. Similarity of the broadband peak non-dimensional frequency ($St_D \equiv fD_j/U_j$) reveals the ability of the model simulations to capture the organized motion in the shear layer. The steeper high frequency spectral decay from the model simulations is probably because of the lack of adequate resolution of fine-scale turbulence features in this coarse computation. Further validation of the mean and unsteady flows were conducted at other locations in the domain, showing qualitative capture of the flow evolution.

SOUND FIELD MODEL RESULTS

The vorticity formulation of the low Mach number acoustic analogy[18] was used to compute the noise generation from the flow field generated by the vortex filament simulations. The right hand side of the inhomogeneous wave equation (see eq. 2) was used to approximate the source distribution and far field acoustics.

$$\frac{\partial^2 p}{\partial t^2} - c^2 \Delta p = -\rho \nabla \bullet (\mathbf{u} \times \boldsymbol{\omega}) = -\rho(-|\boldsymbol{\omega}|^2 - \mathbf{u} \bullet \Delta \mathbf{u}) \qquad (2)$$

The vorticity field is localized in the shear layer, assuming a Gaussian distribution for the vortex core. Thus, source terms could be computed semi-analytically and accurately although a coarse spatial resolution was used for these low order computations. Similar computations of acoustics from finite-difference or other grid-based simulation methods would require very finely resolved spatial fields and computationally intensive procedures. **Figure 4** displays the line noise source distribution along the jet axis as a function of the non-dimensional noise frequency (St_D), after radial and circumferential averaging (used here for simplicity) of the time-retarded source terms from eq. 2 evaluated in the acoustic far field. Such an estimate of the source distribution ignores the three-dimensional aspects of the noise generation.

The experimentally measured source distribution in the jet near field are shown as a function of the non-dimensional noise source frequency (St_D) in **Fig. 5**. The source amplitudes are for a M_j = 0.6

jet issuing into a still ambient and are displayed as sound pressure levels (dB) normalized by the peak noise levels. The trend of low frequency noise source peaks located farther downstream and high frequency sources being dominant upstream are well captured by the vortex model computations. The relatively rapid fall-off of the noise amplitudes at the high frequencies may be attributed to the neglect of fine-scale turbulence effects in these "coarse" vortex model simulations. Furthermore, the coarse spatial resolution of the phased array used for the measurement (namely, one jet diameter) makes the experimental source distributions at high frequencies appear more broadly distributed in the streamwise direction. The far field sound pressure at sideward angles show narrower spectral peaks compared to experimental results (not shown here). Computations of the absolute sound levels indicate much lower levels than measured. This may be due to the use of a low Mach number acoustic analogy for the noise computations and the neglect of three-dimensional sources. Recent application of an O(1) Mach number acoustic analogy for the three-dimensional source field has significantly improved the quantitative agreement and will be reported in an upcoming paper (R. Reba, private communication).

The radial distribution of the sources was also investigated (not shown here), revealing that the sources are localized around the lip line in the shear layer. For low frequencies, the noise sources are distributed in the streamwise direction and located downstream, and for the higher frequencies, the sources are more compact along the jet axis and located farther upstream. The spatiotemporal evolution of the noise sources (namely the time-retarded, spatial average of the right hand side of eq. 2) and the corresponding integrated far-field pressure signature are shown in **Figure 6**, displaying the convected noise sources and their far-field noise. The sources are convected at a wide range of velocities around 60% of the centerline jet velocity. The dominant source (occurring at low frequencies) appears to be associated with unsteady, large-scale vortex mergers and interactions occurring over a wide streamwise extent in the jet shear layer, as observed in prior direct numerical simulations of compressible two-dimensional mixing layers[19]. Note also the two "noisy" temporal events identified in the pressure trace and their corresponding source fields. This role of vortex interaction-generated noise was also verified by simultaneous visualization of the spatiotemporal azimuthal vorticity field of the jet shear layer.

SUMMARY

Reduced order, three-dimensional vortex methods have been used to compute the flow field and noise generation from unsteady flow structures in a jet flow. Comparisons with experimentally measured flow and noise characteristics of a subsonic, turbulent, single stream, cold jet suggest the effectiveness of such simple dynamical models in capturing qualitative features of the jet flow and its noise generation. The present study represents the first attempt to model the broadband sound generation from organized structures in an unsteady, three-dimensional jet flow. Being computationally inexpensive (compared to high-dimensional DNS, LES) while capturing relevant unsteady flow physics (in contrast with RANS-based approaches) makes the use of these methods for understanding and controlling noise generation attractive.

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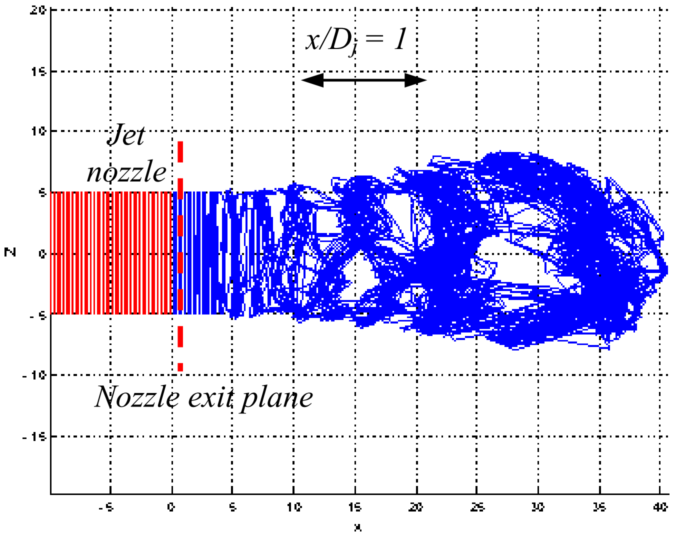

Figure 1. Instantaneous snapshot of three-dimensional vortex filament flow field.

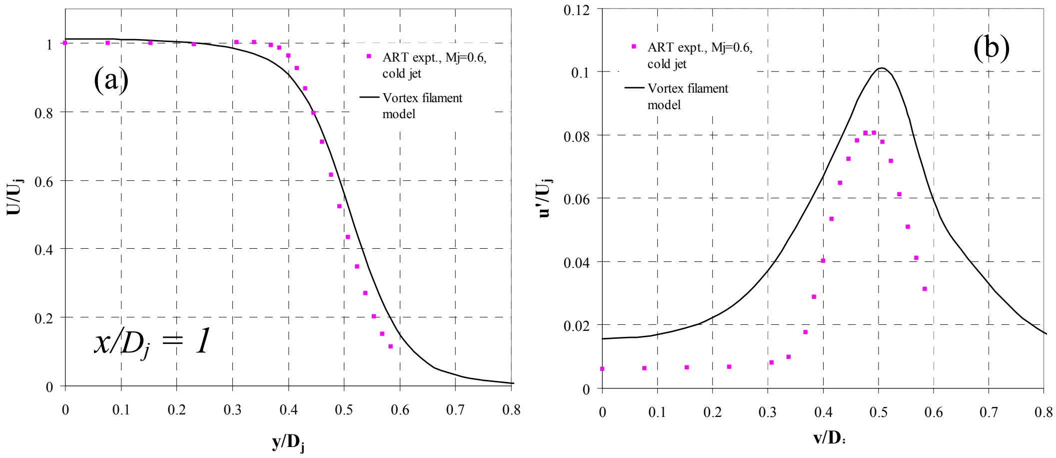

Figure 2. Comparison of longitudinal mean velocity (a) and turbulence intensity (b) profiles between vortex model simulations (solid line) and experimental data (symbols) in a M_j=0.6 single stream jet.

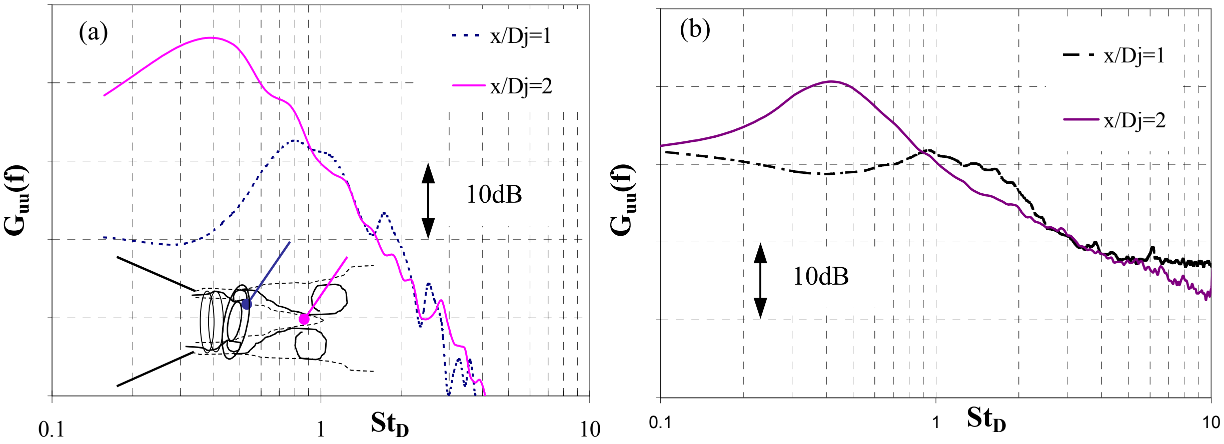

Figure 3. Power spectra of longitudinal velocity component recorded in vortex model (a) and experimentally in a M_j=0.6 jet (b) at x/D_j=1, 2; inset displays the recording location for the velocity traces.

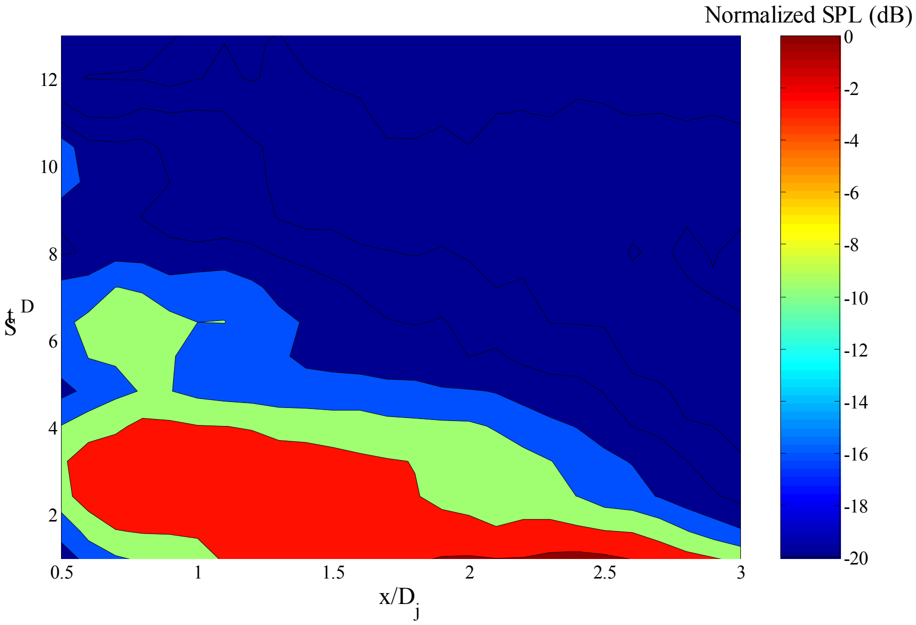

Figure 4. Jet noise source distribution computed from the vortex model.

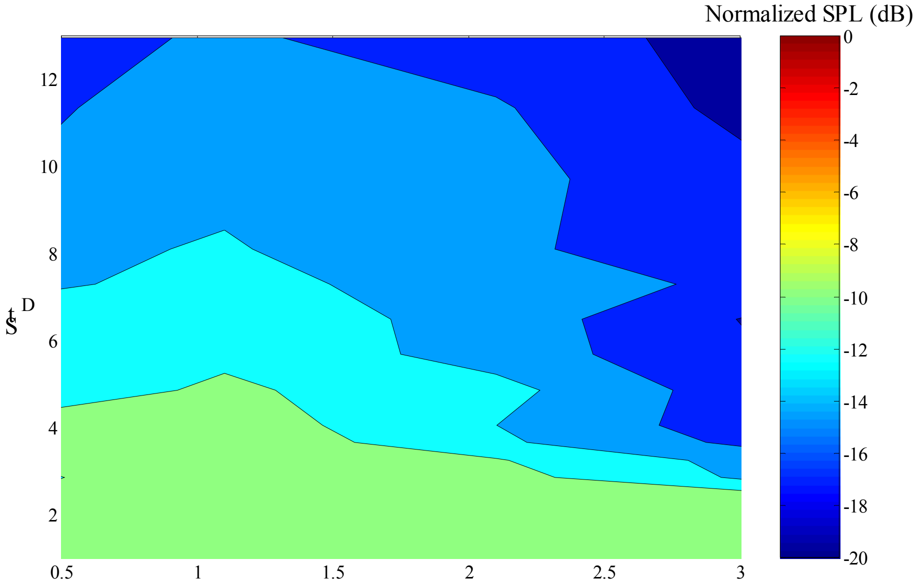

Figure 5. Noise source distribution measured using linear phased array in M_j=0.6 single stream jet.

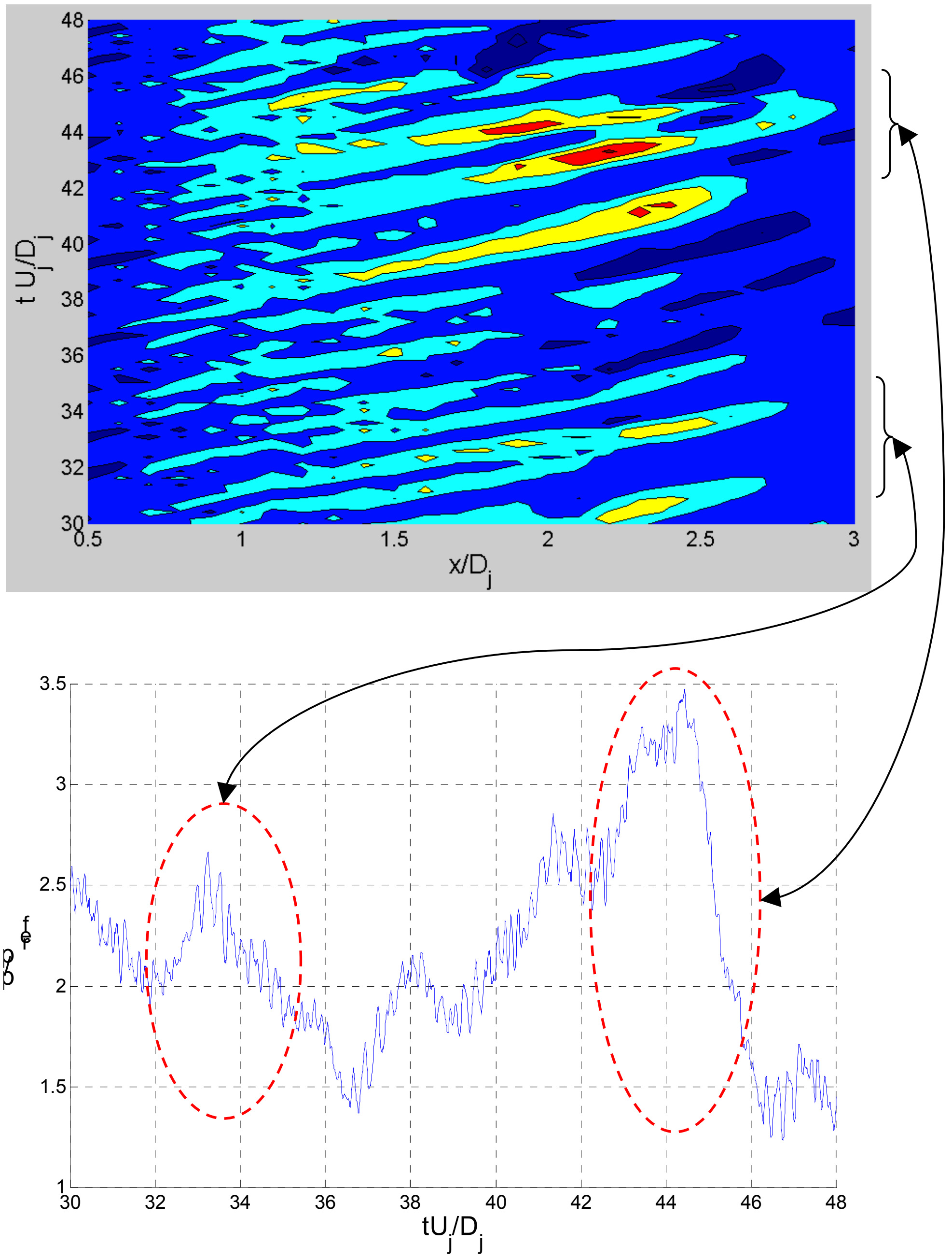

Figure 6. Spatiotemporal evolution of the noise source (a) and its acoustic signature (b); the encircled "events" in the pressure trace correspond to flow interactions occurring over an extended flow domain.