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STABILITY PROPERTIES OF 2D FLOW AROUND AHMED BODY

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Abstract. The global stability analysis for Ahmed Body, representing a generic shape of an automobile, has been performed. The influence of position of the body and Reynolds number on the stability properties (eigenmodes and eigenvalues) has been investigated and the critical Reynolds number has been estimated. Eigenmodes obtained with the analysis provide better understanding of physical phenomena and are intended to be used in modelling and control of the flow.

Key words: Ahmed Body, stability, control, CFD, eigenvalues, subspace iteration

1. Introduction

There are many interesting phenomena appearing in the laminar flow with the growth of Reynolds number. For some value Re_{Cr}, the disturbance introduced into flow, instead of being damped, becomes amplified. The flow becomes unstable and this value of Reynolds number is called critical.

Flow stability analysis is usually performed for simple geometries. The non-parallel flow formulation extends the validity of the analysis to general flow. Global stability analysis delivers not only information about critical Reynolds number, where the flow becomes unstable, but also a set of eigenmodes and eigenvalues, that may be used in low dimensional modelling and control of the flow.

2. Governing Equation

To perform global, linear stability analysis of incompressible flow, steady solution of Navier-Stokes equation (2.1) is required.

$$\dot{V}_i + V_{i,j}V_j + P_{,i} - \frac{1}{Re}V_{i,jj} = 0 \tag{2.1}$$

Assumption, that the unsteady solution V is a sum of steady solution $\bar{V}$ and a small disturbance $\tilde{V}$ (which allows to linearize the equations), leads to formulation of generalized differential eigenvalue problem (2.2)

$$\lambda\tilde{V}_i + \tilde{V}_j\bar{V}_{i,j} + \bar{V}_j\tilde{V}_{i,j} + \tilde{P}_{,i} - \frac{1}{Re}\tilde{V}_{i,jj} = 0 \qquad \text{or} \qquad Ax - \lambda Bx = 0 \tag{2.2}$$

The flow control and stability analysis strongly depends on numerical algorithms. Thus, the proper choice of preconditioning and method of solution is essential. Performed studies [3] indicate, that subspace iteration method, used with Cayley preconditioning (2.3), achieves acceptable performance and convergence rate.

$$((1-\alpha_3)A - (\alpha_1 - \alpha_2\alpha_3)B)x - \mu(A - \alpha_2 B)x = 0 \tag{2.3}$$

Penalty formulation is used to eliminate the pressure term. In the analyses, modified version of program described in [3] has been used.

3. Ahmed Body

It is of great interest to analyse with this approach flow configurations having practical importance.

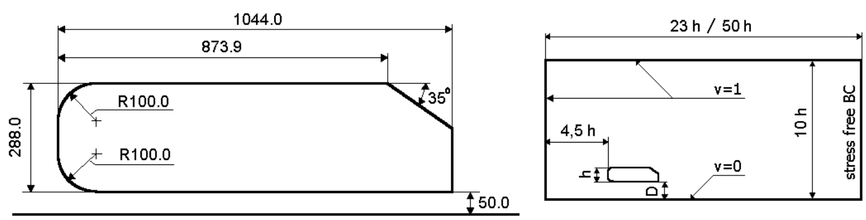

Figure 1. Dimensions of Ahmed Body and computational domain.

One of them is Ahmed Body (Fig. 1), created in 1981 by S.R. Ahmed [1]. It is generic shape of a truck and, although highly simplified, it generates all essential features of flow around car (flow displacement around the nose, uniform flow at the middle and separation and wake at the end of body). This model is widely used during studies on flow processes involved in drag production, as test-bed for CFD software and in other areas.

The rear slant angle has been set up at 35 degrees. The chosen dimensions and boundary conditions have been set as shown on Fig. 1. Comparing to simpler geometries, Ahmed Body wake may be considered as a combination

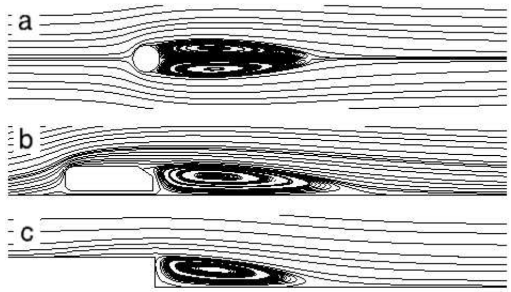

Figure 2. Comparison of geometries: a) cylinder wake b) Ahmed Body c) backward-facing step.

of bluff-body wake (where the distance from object to ground is relatively large) and sudden-expansion channel or backward-facing step (see Fig. 2).

As can be found in [5], the results of 2D computations can be comparable to averaged 3D solution. Additionally, there exist the experimental results of quasi-2D flow around Ahmed body with lengthened spanwise size [2]. Computations presented in this paper are associated with that experiment. These assumptions and the difficulties arising in development of 3D stability solver determined the reduction of geometry to two dimensions.

4. Computation Strategy and used Meshes

The performed numerical experiments consisted of two stages. First, the distance from Ahmed Body to ground D was reduced at constant value of Reynolds number $Re = 100$. At first step, as analogy to bluff-body wake, the object was placed in the middle of the domain (distance $D = 4.5h$). At last step, it was placed in proper distance ($D = 0.17361h$) from ground. In this stage, various unstructured meshes having approximately 3850 nodes and 1850 second-order triangle elements were used.

In second part of analyses, the distance from ground to body was constant, and the Reynolds number (based on height of Ahmed Body) was being changed. Two kinds of meshes were used here: one short-domain grid with 15600 nodes and one lengthened-domain grid with 7513 nodes (see Fig. 3).

5. Results of Computations

5.1. Influence on the distance from ground

When placed in the middle of the domain, the Ahmed Body behaves similarly to circular cylinder. Steady solution and observed eigenmodes (see Fig. 4) are comparable to those appearing in bluff body wake, von Karman street of vertices is also visible.

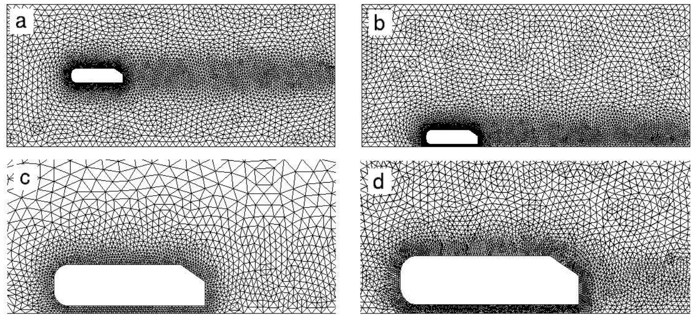

Figure 3. Used meshes: a) $D = 4.5h$ b) $D = 0.17361h$ c) details of low-resolution mesh d) details of high resolution mesh.

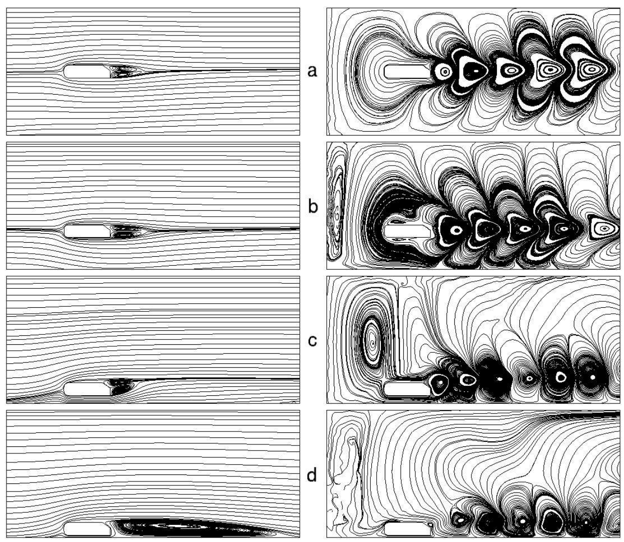

Figure 4. Streamlines of steady solution (left) and first eigenmode pair (right) for $Re = 100$ and varying D: a) $D = 4.5h$, b) $D = 2.5h$, c) $D = 0.66h$, d) $D = 0.17361h$.

This behavior persists until $D = 1.5h$. During further reduction of the distance between the body and the ground the influence of boundary becomes more and more important, and for $D < 0.66h$ the results begin to be similar to backward-facing step solution, with longer separation in steady solution and eigenmodes similar to Kelvin-Helmholtz modes characteristic for the shear-layer modes. Additionally, the real parts of eigenvalues increase with the approaching to the ground.

Critical value of D depends on Reynolds number: the higher Re, the longer the flow pattern remains cylinder-like. For example, for $Re = 50$ the flow pattern changes to step-like near $D = 0.8$ and for $Re = 150$ - near $D = 0.5$

5.2. Influence of Reynolds number

With the growth of Reynolds number, the real parts of the conjugate eigenvalues decrease (see Fig. 5). At $Re > 300$ the first pair of them (5/6) becomes negative. That means that the corresponding eigenmodes are unstable, and therefore the flow is unstable, as well. At $Re = 325$ there are three pair of unstable modes.

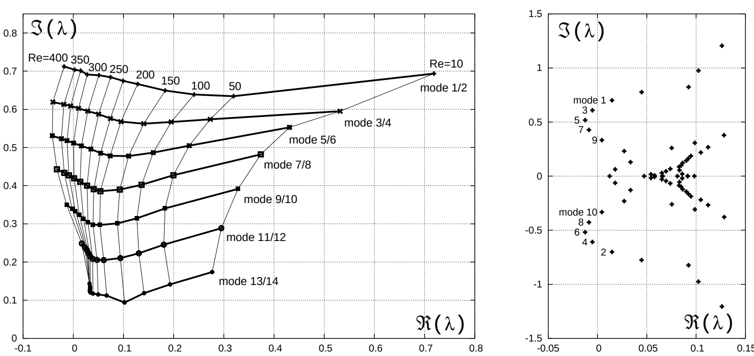

Figure 5. Evolution of eigenvalues with growth of Reynolds number (left) and spectrum for Re=325 (right).

When comparing the spectrum (see Fig. 5) and eigenmodes corresponding to the first eigenvalues (see Fig. 6), the dependence between imaginary parts of eigenvalues, frequency and wave number of the eigenmodes is visible. With the growth of Re, the mentioned values change much slower. This results in similarity of eigenmodes for different Reynolds numbers.

6. Summary

The study of stability of Ahmed Body, performed on different meshes has given very close results. The spectrum of eigenmodes and eigenvalues has been obtained for varying distance from the body to the ground and for varying Reynolds number. The variation of seven pairs of eigenvalues has been traced, and five of the corresponding eigenmodes have been presented for supercritical ($\Re(\lambda) < 0$) Reynolds number ($Re = 325$).

In both stages, although analysed Reynolds numbers were relatively small, many phenomena (like wake, separation, von Karman modes, etc.) have appeared. The separation area for $Re > 400$ crosses the boundary of domain, what can influence the results, therefore for higher Re, the change of mesh is required.

To obtain given eigenmodes from the spectrum, different parameters of Cayley preconditioning were used ($a_1 = -0.1$ / $a_2 = 0.5$, $a_1 = 0.15$ / $a_2 = 0.8$, etc.). The found eigenmodes may be used in low dimensional approximation

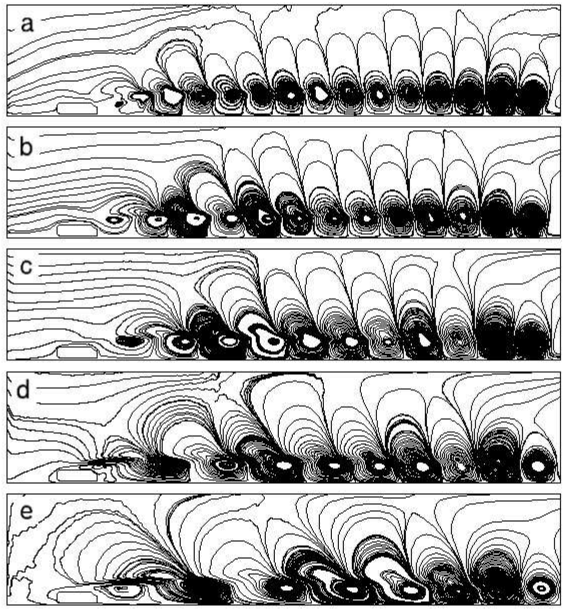

Figure 6. First five eigenmode pairs for $Re = 325$; a) $\lambda_{1/2} = 0.0177 \pm 0.7012\imath$ b) $\lambda_{3/4} = -0.0051 \pm 0.6097\imath$ c) $\lambda_{5/6} = -0.0125 \pm 0.5181\imath$ d) $\lambda_{7/8} = -0.0087 \pm 0.4272\imath$ e) $\lambda_{9/10} = 0.0045 \pm 0.3326\imath$.

of the flow [4]. This, and three-dimensional stability analysis, are the areas of future interest.

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